

1)

$$4^x \cdot 25^x = 0,01 \cdot (10^{3x-1})^2$$

$$100^x = 10^{-2+6x-2}$$

$$10^{2x} = 10^{6x-4}$$

$$2x = 6x - 4$$

$$x = 1$$

$$\underline{\underline{K = \{ 1 \}}}$$

2)

$$\left(\frac{5}{3}\right)^{x+1} \cdot \left(\frac{9}{25}\right)^{x^2+2x-11} = \left(\frac{125}{27}\right)^3$$

$$\left(\frac{5}{3}\right)^{x+1} \cdot \left(\frac{5}{3}\right)^{-2(x^2+2x-11)} = \left(\frac{5}{3}\right)^9$$

$$\left(\frac{5}{3}\right)^{x+1-2x^2-4x+22} = \left(\frac{5}{3}\right)^9$$

$$-2x^2 - 3x + 23 = 9$$

$$2x^2 + 3x - 14 = 0$$

$$D = 9 + 112 = 121$$

$$\sqrt{D} = 11$$

$$x_1 = \frac{-3+11}{4} = 2$$

$$x_2 = \frac{-3-11}{4} = -\frac{7}{2}$$

$$\underline{\underline{K = \left\{ 2; -\frac{7}{2} \right\}}}$$

3)

$$3^{2-x} = 5^{x-2}$$

$$9 \cdot \left(\frac{1}{3}\right)^x = \frac{1}{25} \cdot 5^x \quad / \cdot 3^x \quad / \cdot 25$$

$$225 = 5^x \cdot 3^x$$

$$15^2 = 15^x$$

$$x = 2$$

$$\underline{\underline{K = \{ 2 \}}}$$

4)

$$(0,25)^{2-x} = \frac{256}{2^{x+3}}$$

$$2^{-2(2-x)} = 2^8 \cdot 2^{-x-3}$$

$$2^{-4+2x} = 2^{5-x}$$

$$-4 + 2x = 5 - x$$

$$x = 3$$

$$\underline{\underline{K = \{ 3 \}}}$$

5)

$$\frac{0,2^{x+0,5}}{\sqrt{5}} = \frac{0,04^x}{25}$$

$$5^{-1\left(x+\frac{1}{2}\right)} \cdot 5^{-\frac{1}{2}} = 5^{-2x} \cdot 5^{-2}$$

$$5^{-x-\frac{1}{2}-\frac{1}{2}} = 5^{-2x-2}$$

$$-x-1 = -2x-2$$

$$x = -1$$

$$\underline{\underline{K = \{ -1 \}}}$$

6)

$$\left(\frac{4}{9}\right)^x \cdot \left(\frac{27}{8}\right)^{x-1} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{2x} \cdot \left(\frac{2}{3}\right)^{-3(x-1)} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{2x-3x+3} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{3-x} = \left(\frac{2}{3}\right)^1$$

$$3-x=1$$

$$x=2$$

$$\underline{\underline{K = \{ 2 \}}}$$

7)

$$\frac{6^{x^2}}{2^{-15}} = \frac{3^{-15}}{6^{12-12x}} \quad / \cdot 2^{-15} \quad / \cdot 6^{12-12x}$$

$$6^{x^2} \cdot 6^{12-12x} = 3^{-15} \cdot 2^{-15}$$

$$6^{x^2-12x+12} = 6^{-15}$$

$$x^2 - 12x + 12 = -15$$

$$x^2 - 12x + 27 = 0$$

$$(x-9)(x-3) = 0$$

$$x_1 = 9 \vee x_2 = 3$$

$$\underline{\underline{K = \{ 3; 9 \}}}$$

8)

$$x^{-1}\sqrt[3]{9^x} \cdot \sqrt[3]{3^{x-3}} = 27 \quad D: x-1 \in \mathbb{Z}^+ \wedge x \in \mathbb{Z}^+$$

$$3^{\frac{2x}{x-1} + \frac{x-3}{x}} = 3^3$$

$$\frac{2x^2 + x^2 - 4x + 3}{x(x-1)} = 3$$

$$3x^2 - 4x + 3 = 3x^2 - 3x$$

$$x = 3 \in D$$

$$\underline{\underline{K = \{ 3 \}}}$$

9)

$$4^{\sqrt{x+1}} = 64 \cdot 2\sqrt{x+1} \quad x+1 > 0 \Rightarrow D: x > -1$$

$$2^{2\sqrt{x+1}} = 2^{6+\sqrt{x+1}}$$

$$2\sqrt{x+1} = 6 + \sqrt{x+1}$$

$$\sqrt{x+1} = 6 \quad / R^2$$

$$x+1 = 36$$

$$x = 35 \in D$$

Zk :

$$L: 4^6 = 2^{12}$$

$$P: 64 \cdot 2^6 = 2^{12}$$

$$L = P$$

$$\underline{\underline{K = \{ 35 \}}}$$

10)

$$\frac{2^{2x-1} \cdot 4^{x+1}}{8^{x-1}} = 64$$

$$\frac{2^{2x-1} \cdot 2^{2(x+1)}}{2^{3(x-1)}} = 2^6$$

$$2^{2x-1+2x+2-3x+3} = 2^6$$

$$x+4 = 6$$

$$x = 2$$

$$\underline{\underline{K = \{ 2 \}}}$$

11)

$$\left[2.\left(2^{\sqrt{x}+3} \right)^{\frac{1}{2\sqrt{x}}} \right]^{\frac{2}{\sqrt{x}}-1} = 4 \quad x > 0; \ x \neq 1 \Rightarrow D : (0;1) \cup (1; \infty)$$

$$\left(2.2^{\frac{\sqrt{x}+3}{2\sqrt{x}}} \right)^{\frac{2}{\sqrt{x}-1}} = 2^2$$

$$\left(2^{1+\frac{\sqrt{x}+3}{2\sqrt{x}}} \right)^{\frac{2}{\sqrt{x}-1}} = 2^2$$

$$2^{\frac{3\sqrt{x}+3}{2\sqrt{x}} \cdot \frac{2}{\sqrt{x}-1}} = 2$$

$$\frac{3\sqrt{x}+3}{x-\sqrt{x}} = 2$$

$$3\sqrt{x}+3 = 2x-2\sqrt{x}$$

$$2x-3 = 5\sqrt{x} \ / \ R^2$$

$$4x^2 - 37x + 9 = 0$$

$$D = 1369 - 144 = 1225$$

$$\sqrt{D} = 35$$

$$x_1 = \frac{37+35}{8} = 9$$

$$x_2 = \frac{37-35}{8} = \frac{1}{4}$$

ZK : (9)

$$L : \left[2.\left(2^{\sqrt{9}+3} \right)^{\frac{1}{2\sqrt{9}}} \right]^{\frac{2}{\sqrt{9}-1}} = \left[2.\left(2^{3+3} \right)^{\frac{1}{2.3}} \right]^{\frac{2}{3-1}} = \left[\left(2.2^6 \right)^{\frac{1}{6}} \right]^{\frac{2}{2}} = (2.2)^1 = 4$$

$$P : 4$$

$$L = P$$

$$ZK : \left(\frac{1}{4} \right)$$

$$L : 2^{-18}$$

$$P : 4$$

$$L \neq P$$

$$\underline{\underline{K = \{ 9 \}}}$$

12)

$$4 \cdot 3^{x+1} - 3^{x-1} = 315$$

$$12 \cdot 3^x - \frac{1}{3} \cdot 3^x = 315 \quad / \cdot 3$$

$$36 \cdot 3^x - 3^x = 315 \cdot 3$$

$$35 \cdot 3^x = 315 \cdot 3 \quad / : 35$$

$$3^x = 9 \cdot 3$$

$$3^x = 3^3$$

$$x = 3$$

$$\underline{\underline{K = \{ 3 \}}}$$

13)

$$2^{x-1} + 2^{x-2} + 2^{x-3} = 448$$

$$\frac{1}{2} \cdot 2^x + \frac{1}{4} \cdot 2^x + \frac{1}{8} \cdot 2^x = 448 \quad / \cdot 8$$

$$4 \cdot 2^x + 2 \cdot 2^x + 2^x = 448 \cdot 8$$

$$7 \cdot 2^x = 448 \cdot 8 \quad / : 7$$

$$2^x = 64 \cdot 8$$

$$2^x = 2^9$$

$$x = 9$$

$$\underline{\underline{K = \{ 9 \}}}$$

14)

$$3^{2x-1} + 3^{2x-2} - 3^{2x-4} = 315$$

$$\frac{1}{3} \cdot 3^{2x} + \frac{1}{9} \cdot 3^{2x} - \frac{1}{81} \cdot 3^{2x} = 315 \quad / \cdot 81$$

$$27 \cdot 3^{2x} + 9 \cdot 3^{2x} - 3^{2x} = 315 \cdot 81$$

$$35 \cdot 3^{2x} = 315 \cdot 81 \quad / : 35$$

$$3^{2x} = 9 \cdot 81$$

$$3^{2x} = 3^6$$

$$2x = 6$$

$$x = 3$$

$$\underline{\underline{K = \{ 3 \}}}$$

15)

$$5 \cdot 9^x - \frac{8}{3} \cdot 12^x = 3^x \cdot 4^{x+1}$$

$$5 \cdot 3^{2x} - \frac{8}{3} \cdot 3^x \cdot 4^x = 3^x \cdot 4 \cdot 4^x \quad / : 3^x \quad / : 3$$

$$15 \cdot 3^x - 8 \cdot 4^x = 12 \cdot 4^x$$

$$15 \cdot 3^x = 20 \cdot 4^x$$

$$\frac{3^x}{4^x} = \frac{20}{15}$$

$$\left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^{-1}$$

$$x = -1$$

$$\underline{\underline{K = \{ -1 \}}}$$

16)

$$3 \cdot 4^x + \frac{9^{x+2}}{3} = 6 \cdot 4^{x+1} - \frac{9^{x+1}}{2}$$

$$3 \cdot 4^x + \frac{81}{3} \cdot 9^x = 24 \cdot 4^x - \frac{9}{2} \cdot 9^x$$

$$27 \cdot 9^x + \frac{9}{2} \cdot 9^x = 24 \cdot 4^x - 3 \cdot 4^x$$

$$\frac{63}{2} \cdot 9^x = 21 \cdot 4^x \quad / : 4^x \quad / \cdot \frac{2}{63}$$

$$\left(\frac{3}{2}\right)^{2x} = \left(\frac{3}{2}\right)^{-1}$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$\underline{\underline{K = \left\{ -\frac{1}{2} \right\}}}$$

17)

$$4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$$

$$2^{2x} - \frac{1}{\sqrt{3}} \cdot 3^x = \sqrt{3} \cdot 3^x - \frac{1}{2} \cdot 2^{2x}$$

$$2^{2x} + \frac{1}{2} \cdot 2^{2x} = \sqrt{3} \cdot 3^x + \frac{1}{\sqrt{3}} \cdot 3^x$$

$$\frac{3}{2} \cdot 2^{2x} = \frac{3+1}{\sqrt{3}} \cdot 3^x \quad / : 3^x \quad / \cdot \frac{2}{3}$$

$$\frac{2^{2x}}{3^x} = \frac{8}{3 \cdot \sqrt{3}}$$

$$\frac{4^x}{3^x} = \frac{2^3}{3^{\frac{3}{2}}}$$

$$\left(\frac{4}{3}\right)^x = \frac{4^{\frac{3}{2}}}{3^{\frac{3}{2}}}$$

$$\left(\frac{4}{3}\right)^x = \left(\frac{4}{3}\right)^{\frac{3}{2}}$$

$$x = \frac{3}{2}$$

$$\underline{\underline{K = \left\{ \frac{3}{2} \right\}}}$$

18)

$$6^{\frac{x}{2}} - 5^{\frac{x}{2}} = 6^{\frac{x}{2}-1}$$

$$6^{\frac{x}{2}} - \frac{1}{6} \cdot 6^{\frac{x}{2}} = 5^{\frac{x}{2}}$$

$$\frac{5}{6} \cdot 6^{\frac{x}{2}} = 5^{\frac{x}{2}} \quad / : 5^{\frac{x}{2}} \quad / \cdot \frac{6}{5}$$

$$\frac{6^{\frac{x}{2}}}{5^{\frac{x}{2}}} = \frac{6}{5}$$

$$\left(\frac{6}{5}\right)^{\frac{x}{2}} = \left(\frac{6}{5}\right)^1$$

$$\frac{x}{2} = 1$$

$$x = 2$$

$$\underline{\underline{K = \{ 2 \}}}$$

19)

$$\frac{1}{4} \cdot 2^x + \frac{1}{2} \cdot 4^x = 9 \quad / \cdot 4$$

$$2^x + 2 \cdot 2^{2x} - 36 = 0$$

$$\text{substituce: } a = 2^x$$

$$2 \cdot a^2 + a - 36 = 0$$

$$D: 1 + 4 \cdot 2 \cdot 36 = 289$$

$$\sqrt{D}: 17$$

$$a_1 = \frac{-1 + 17}{4} = 4$$

$$a_2 = \frac{-1 - 17}{4} = -\frac{9}{2}$$

dosadíme zpátky do substituce:

$$2^x = a_1$$

$$2^x = 4$$

$$2^x = 2^2$$

$$x_1 = 2$$

$$2^x = a_2$$

$$2^x = -\frac{9}{2}$$

$$x_2 = \{ \} \quad \text{rovnice nemá řešení}$$

$$\underline{\underline{K = \{ 2 \}}}$$

20)

$$3^{2+x} + 3^{4-x} = 90$$

$$3^2 \cdot 3^x + 3^4 \cdot 3^{-x} = 90$$

$$9 \cdot 3^x + 81 \cdot \frac{1}{3^x} = 90 \quad / \cdot 3^x$$

$$9 \cdot 3^{2x} - 90 \cdot 3^x + 81 = 0 \quad / : 9$$

$$3^{2x} - 10 \cdot 3^x + 9 = 0$$

$$\text{substitute: } a = 3^x$$

$$a^2 - 10a + 9 = 0$$

$$(a-1)(a-9) = 0$$

$$a_1 = 1 \vee a_2 = 9$$

dosadíme zpátky do substitute:

$$3^x = 1$$

$$3^x = 3^0$$

$$x_1 = 0$$

$$3^x = 9$$

$$3^x = 3^2$$

$$x_2 = 2$$

$$\underline{\underline{K = \{ 0; 2 \}}}$$
