LOGARITMICKÉ ROVNICE – ŘEŠENÉ PŘÍKLADY

Příklad 1

$\log(2x+10) = 2\log(x+1)$ $\quad$ *podmínka* : $2x+10>0 \quad \wedge \quad x+1>0$

$\log(2x+10) = \log(x+1)^2$ $\quad$ $x>-5 \quad \wedge \quad x>-1$

$2x+10 = (x+1)^2$ $\quad$ $D: x>-1$

$2x+10 = x^2+2x+1$

$x^2 = 9$

$|x| = 3$

$x_1 = 3 \in D \vee \; x_2 = -3 \notin D$

$\underline{\underline{K = \{3\}}}$

Příklad 2

$\log x = 2 - \log 5$ $\quad$ *podmínka* : $x>0$

$\log x = \log 100 - \log 5$

$\log x = \log \frac{100}{5}$

$x = \frac{100}{5}$

$x = 20 \in D$

$\underline{\underline{K = \{20\}}}$

Příklad 3

$\log(x+1) + \log(x-1) - \log(x-2) = \log 8$ $\quad$ *podm.*: $x>-1 \; \wedge \; x>1 \; \wedge \; x>2$

$\log \frac{(x+1)(x-1)}{x-2} = \log 8$ $\quad$ $D: x>2$

$\frac{x^2-1}{x-2} = 8$

$x^2 - 1 = 8x - 16$

$x^2 - 8x + 15 = 0$

$(x-5)(x-3) = 0$

$x_1 = 5 \in D \; \vee \; x_2 = 3 \in D$

$\underline{\underline{K = \{3;5\}}}$

Příklad 4

$\log_3(x+1)+\log_3(x+3)=1$ $\qquad$ *podm.*: $x>-1 \ \wedge \ x>-3$

$\log_3((x+1)+\log_3(x+3)=\log_3 3$ $\qquad$ $D: x>-1$

$\log_3(x+1)(x+3)=\log_3 3$

$(x+1)(x+3)=3$

$x^2+4x+3=3$

$x^2+4x=0$

$x(x+4)=0$

$x_1=0\in D \ \vee \ x_2=-4\notin D$

$\underline{\underline{K=\{0\}}}$

Příklad 5

$\log_3 x+\log_3(x+1)=2-\log_3\frac{3}{2}$ $\qquad$ *podm.*: $x>0 \ \wedge \ x>-1$

$\log_3 x+\log_3(x+1)=2.\log_3 3-\log_3\frac{3}{2}$ $\qquad$ $D: x>0$

$\log_3 x(x+1)=\log_3\frac{3^2}{\frac{3}{2}}$

$x^2+x=\frac{9}{\frac{3}{2}}$

$x^2+x=6$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x_1=-3\notin D \ \vee \ x_2=2\in D$

$\underline{\underline{K=\{2\}}}$

Příklad 6

$\log x^2+\log\sqrt{x}-3\log x=1$ $\qquad$ $D: x>0$

$2\log x+\log x^{\frac{1}{2}}-3\log x=1$

$\frac{1}{2}\log x-\log x=1$

$-\frac{1}{2}\log x=1 \ / .(-2)$

$\log x=-2 \ \Leftrightarrow \ 10^{-2}=x$

$x=0{,}01\in D$

$\underline{\underline{K=\{0{,}01\}}}$

Příklad 7

$5\log\sqrt[3]{x}-4\log\sqrt[6]{x}+\frac{1}{2}\log x^8=5-\log x^5 \qquad D: x>0$

$5\log x^{\frac{1}{3}}-4\log x^{\frac{1}{6}}+\frac{1}{2}.8\log x=5-5\log x$

$\frac{5}{3}\log x-\frac{2}{3}\log x+4\log x+5\log x=5$

$10\log x=5 \quad /:10$

$\log x=\frac{1}{2} \Leftrightarrow 10^{\frac{1}{2}}=x$

$x=\sqrt{10}\in D$

$\underline{\underline{K=\left\{\sqrt{10}\right\}}}$

Příklad 8

$2\log_3\frac{x-3}{x-7}+1=\log_3\frac{x-3}{x-1} \qquad podm.: \frac{x-3}{x-7}>0 \ \wedge \ \frac{x-3}{x-1}>0$

$\log_3\left(\frac{x-3}{x-7}\right)^2+\log_3 3=\log_3\frac{x-3}{x-1} \qquad D: x\in(-\infty;1)\cup(7;\infty)$

$\log_3 3.\left(\frac{x-3}{x-7}\right)^2=\log_3\frac{x-3}{x-1}$

$3.\left(\frac{x-3}{x-7}\right)^2=\frac{x-3}{x-1} \quad /.(x-7)^2.(x-1)$

$3.(x-3)^2.(x-1)=(x-3)(x-7)^2 \qquad /:(x-3)$

$3.(x-3).(x-1)=x-7)^2$

$3x^2-12x+9=x^2-14x+49 \quad /:2$

$x^2+x-20=0$

$(x+5)(x-4)=0$

$x_1=-5\in D \ \vee \ x_2=4\notin D$

$\underline{\underline{K=\{-5\}}}$

Příklad 9

$\log_5 \sqrt{3x-2} + \log_5 \sqrt{4x-7} = \log_5 13$ $\quad$ *podm.*: $x > \frac{2}{3} \ \wedge \ x > \frac{7}{4}$

$\frac{1}{2}\log_5 (3x-2) + \frac{1}{2}\log_5 (4x-7) = \log_5 13 \quad / .2$ $\quad$ $D: x > \frac{7}{4}$

$\log_5 (3x-2)(4x-7) = 2.\log_5 13$

$(3x-2)(4x-7) = 13^2$

$12x^2 - 29x + 28 = 169$

$12x^2 - 29x - 155 = 0$

$D = 841 + 7440 = 8281$

$\sqrt{D} = 91$

$x_1 = \frac{29+91}{24} = \frac{120}{5} = 5 \in D$

$x_2 = \frac{29-91}{24} = \frac{-31}{24} \notin D$

$\underline{\underline{K = \{5\}}}$

Příklad 10

$\frac{3+\log x}{2-\log x} = 4 \quad / .(2-\log)$ $\quad$ *podm.*: $x > 0 \ \wedge 2 - \log x \neq 0$

$3 + \log x = 4(2 - \log x)$ $\quad$ $\log x \neq 2$

$3 + \log x = 8 - 4\log x$ $\quad$ $x \neq 100$

$\log x + 4\log x = 8 - 3$ $\quad$ $D: x \in (0;100) \cup (100;\infty)$

$5\log x = 5$

$\log x = 1 \ \Leftrightarrow 10^1 = x$

$x = 10 \in D$

$\underline{\underline{K = \{10\}}}$

Příklad 11

$\frac{\log x + 1}{2 + \log x} + \frac{2\log x - 1}{\log x} = 3 \ / .(2+\log x).\log x$ $\quad$ *podm.*: $x > 0 \ \wedge \ \log x \neq 0 \ \wedge \ 2 + \log x \neq 0$

$(\log x + 1)\log x + (2\log x - 1)(2 + \log x) = 3\log x(2 + \log x)$ $\quad$ $x \neq 1$ $\quad$ $\log x \neq -2$

$\log^2 x + \log x + 4\log x - 2 + 2\log^2 x - \log x = 6\log x + 3\log^2 x$ $\quad$ $x \neq 0{,}01$

$2\log x = -2$ $\quad$ $D: x \in (0;0{,}01) \cup (0{,}01;1) \cup (1;\infty)$

$\log x = -1 \ \Leftrightarrow \ 10^{-1} = x$

$x = 0{,}1 \in D$

$\underline{\underline{K = \{0{,}1\}}}$

Příklad 12

$\log x - \frac{4}{\log x} = 0 \qquad / . \log x \qquad\qquad podm.: \; x > 0 \;\wedge\; \log x \neq 0$

$\log^2 x - 4 = 0 \qquad\qquad x \neq 1$

$\log^2 x = 4 \qquad\qquad D: \; x \in (0;1) \cup (1;\infty)$

$|\log x| = 2$

$\log x = \pm 2$

$\log x_1 = 2 \;\Rightarrow\; 10^2 = x_1 \;\Rightarrow\; x_1 = 100 \;\in D$

$\log x_2 = -2 \;\Rightarrow 10^{-2} = x_2 \;\Rightarrow\; x_2 = 0{,}01 \in D$

$\underline{\underline{K = \{0{,}01;100\}}}$

Příklad 13

$\log x^{2\log\sqrt{x}} + \log\frac{1}{x^2} = 3 \qquad\qquad D: x > 0$

$\log x^{2\log\sqrt{x}} + \log x^{-2} = 3$

$2\log\sqrt{x} . \log x - 2\log x = 3$

$2 . \log x^{\frac{1}{2}} . \log x - 2 . \log x = 3$

$2 . \frac{1}{2} \log x . \log x - 2 . \log x = 3$

$\log^2 x - 2\log x - 3 = 0$

$substituce: \; \log x = a$

$a^2 - 2a - 3 = 0$

$(a-3)(a+1) = 0$

$a_1 = 3 \;\vee\; a_2 = -1$

$\log x = a_1$	$\log x = a_2$
$\log x = 3$	$\log x = -1$
$x_1 = 1000 \in D$	$x_2 = 0{,}1 \in D$

$\underline{\underline{K = \{0{,}1;1000\}}}$

Příklad 14

$x^{1+\log x} = 100 \qquad / \qquad D: x > 0$

$(1 + \log x).\log x = \log 100$

$\log x + \log^2 x = 2$

$\log^2 x + \log x - 2 = 0$

$(\log x - 1)(\log x + 2) = 0$

$\log x_1 = 1 \qquad \vee \qquad \log x_2 = -2$

$10^1 = x_1 \qquad\qquad 10^{-2} = x_2$

$x_1 = 10 \in D \qquad\qquad x_2 = 0{,}01 \in D$

$\underline{\underline{K = \{0{,}01;\ 10\}}}$

Příklad 14

$x^{\log x} = \sqrt[4]{10} \qquad / \log R \qquad D: x > 0$

$\log x.\log x = \frac{1}{4}\log 10$

$\log^2 x = \frac{1}{4}$

$|\log x| = \frac{1}{2}$

$\log x_1 = \frac{1}{2} \qquad \vee \qquad \log x_2 = -\frac{1}{2}$

$10^{\frac{1}{2}} = x_1 \qquad\qquad 10^{-\frac{1}{2}} = x_2$

$x_1 = \sqrt{10} \in D \qquad\qquad x_2 = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \in D$

$\underline{\underline{K = \left\{\sqrt{10};\ \frac{\sqrt{10}}{10}\right\}}}$

Příklad 15

$\sqrt{x}^{\log^2 x-1} = 1 \qquad D: x > 0$

$x^{\frac{1}{2}(\log^2 x-1)} = 1 \qquad / \log R$

$\frac{1}{2}(\log^2 x - 1).\log x = \log 1$

$\frac{1}{2}\log^3 x - \frac{1}{2}\log x = \log 1 \quad / .2$

$\log^3 x - \log x = 0$

$\log x(\log^2 x - 1) = 0$

$\log x = 0 \quad \vee \quad \log^2 x - 1 = 0$

$10^0 = x_1 \qquad \log^2 x = 1$

$\underline{x_1 = 1 \in D} \qquad |\log x| = 1$

$\log x = \pm 1$

$\log x = 1 \quad \vee \quad \log x = -1$

$10^1 = x \qquad 10^{-1} = x$

$\underline{x_2 = 10 \in D} \qquad \underline{x_3 = 0{,}1 \in D}$

$\underline{\underline{K = \{0{,}1;\ 1;\ 10\}}}$

Příklad 16

$2^x = 3 \quad / \log R$

$x.\log 2 = \log 3$

$x = \frac{\log 3}{\log 2}$

$x = \frac{0{,}47712}{0{,}30103}$

$x = 1{,}58496$

$\underline{\underline{K = \{1{,}58496\}}}$

Příklad 17

$7^{3x+1} = 14$

$7^{3x}.7 = 14 \quad / : 7$

$7^{3x} = 2 \qquad / \log R$

$3x.\log 7 = \log 2$

$x = \dfrac{\log 2}{3.\log 7}$

$x = \dfrac{0{,}30103}{3.0{,}845098}$

$x = 0{,}118736$

$\underline{\underline{K = \{0{,}118736\}}}$

Příklad 18

$6.5^x = 4.3^x$

$\dfrac{5^x}{3^x} = \dfrac{4}{6}$

$\left(\dfrac{5}{3}\right)^x = \dfrac{2}{3} \qquad / \log R$

$x.\log\dfrac{5}{3} = \log\dfrac{2}{3}$

$x = \dfrac{\log\dfrac{2}{3}}{\log\dfrac{5}{3}}$

$x = \dfrac{-0{,}17609}{0{,}221849}$

$x = -0{,}79373$

$\underline{\underline{K = \{-0{,}79373\}}}$

Příklad 19

$$\frac{2^{x+2}}{2^{1-x}} = \frac{\log 4}{\log 2}$$

$$2^{x+2-(1-x)} = \frac{\log 2^2}{\log 2}$$

$$2^{2x+1} = \frac{2.\log 2}{\log 2}$$

$$2^{2x+1} = 2$$

$$2x+1 = 1$$

$$2x = 0$$

$$x = 0$$

$$\underline{\underline{K = \{0\}}}$$

Příklad 20

$$\frac{5^{3x-1}}{5^{2-x}} = \frac{\log 32}{\log 2}$$

$$5^{3x-1-2+x} = \frac{\log 2^5}{\log 2}$$

$$5^{4x-3} = \frac{5.\log 2}{\log 2}$$

$$5^{4x-3} = 5$$

$$4x-3 = 1$$

$$4x = 4$$

$$x = 1$$

$$\underline{\underline{K = \{1\}}}$$